

Recent progress in wormhole dynamics

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Abstract

Space-time wormholes were introduced in Wheeler's idea of space-time foam. Traversable wormholes as defined by Morris & Thorne became popular as potential short cuts across the universe and even time machines. More recently, the author proposed a general theory of wormhole dynamics, unified with black-hole dynamics. This article gives a brief review of the above ideas and summarizes progress on wormhole dynamics in the last year. Firstly, a numerical study of dynamical perturbations of the first Morris-Thorne wormhole showed it to be unstable, either collapsing to a black hole or exploding to an inflationary universe. This provides a mechanism for inflating a wormhole from space-time foam to usable size. Intriguing critical behaviour was also discovered. Secondly, a wormhole solution supported by pure radiation was discovered and used to find analytic examples of dynamic wormhole processes which were also recently found in a two-dimensional dilaton gravity model: the construction of a traversible wormhole from a Schwarzschild black hole and vice versa, and the enlargement or reduction of the wormhole.

1 Introduction

Space-time wormholes were first described as such by Wheeler [1], who envisaged the still-popular space-time foam: the smooth space-time of General Relativity suffering quantum-gravitational fluctuations in topology at the Planck scale, becoming a continual foam of transient interconnections. Curiously, all the standard black-hole solutions have a wormhole spatial topology, with a minimal surface connecting two asymptotically flat regions, our universe and a mirrored universe. For the simplest Schwarzschild black hole, this Einstein-Rosen bridge was properly understood only much later, also by Wheeler, as a non-traversable wormhole: the two universes are not in causal contact, with any attempted crossing leading only into the black hole. (See last year's proceedings [2] for space-time diagrams and explanations mostly not repeated here). Later still, Morris & Thorne [3] proposed traversible wormholes, which have similar spatial geometry, with a minimal surface connecting two asymptotically flat regions, but such that the minimal surface (the wormhole throat) is preserved in time. Thus a traveller can cross between the two universes at will. Such wormholes became popular both in science fiction and as a research topic [4], apparently also allowing time-machine construction. However, most research has concerned static or cut-and-paste wormholes, or lacked an independently defined exotic matter model. In contrast, the dynamical behaviour of wormholes has received little attention until recently, and it is only in the last year or two that exact solutions describing wormhole construction and enlargement have been found.

2 Static and dynamic wormholes

Morris & Thorne's unusually accessible article gave a list of criteria for traversible wormholes which is still worth consulting. Paraphrasing briefly, the basic wormhole criteria were: (1) a static, spherically symmetric space-time; (2) the Einstein equation; (3) a throat (minimal surface) connecting two asymptotically flat regions; and (4) no (Killing) horizon, so that the wormhole allows two-way travel. The usability criteria were (5) small tidal gravitational forces and (6) small proper time for crossing, by human scales. Finally, the criteria which were mentioned but largely left open were: (7) physically reasonable matter; (8) perturbative stability, e.g. due to a spaceship; and (9) that it should be possible to assemble the wormhole. These are, of course, the more interesting issues.

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Recent work can be summarized by a corresponding list. (1) One needs to generalize to any space-time, without symmetry, in particular to non-static space-times. (2) Although Einstein gravity will be assumed in this article, many alternative gravitational theories allow traversible wormhole solutions, including scalar-tensor theories and brane-world models. The most fundamental point is that one needs to (3) generalize and localize the definition of the throat and (4) impose local two-way traversibility. Such a definition was given by the author [5] in terms of trapping horizons, which are hypersurfaces foliated by marginal surfaces, which are extremal surfaces in a null (light-like) hypersurface [6]–[8]. They can be locally classified as future or past and outer or inner. Examples include the outer and possible inner horizons of black and white holes. For a Morris-Thorne wormhole, the throat is a double outer trapping horizon; double since the minimal surfaces are extremal in both null directions, and outer since the extremal surface should be minimal rather than maximal, encoding the so-called flare-out condition. The doubled nature of the throat indicates that a non-static wormhole will generally have two mouths, defined by outer trapping horizons, connected by a tunnel of trapped surfaces. In the static case, the tunnel shrinks away and the mouths coincide as the throat. A similar understanding was reached contemporaneously by Hochberg & Visser [9], though their flare-out condition can generically select maximal rather than minimal surfaces in a time-symmetric hypersurface.

The author proposed that a black hole can be locally characterized by an achronal (spatial or null) future outer trapping horizon, and a traversible wormhole by two temporal (time-like) outer trapping horizons in mutual causal contact [5]. Note that the main difference is the causal nature of the horizon, which is respectively one-way or two-way traversible, as expected in each case. The Einstein equation then shows that they occur respectively under positive and negative energy density, specifically referring to the null energy condition [5]. This means that they are supported respectively by normal matter or vacuum, and what was dubbed exotic matter [3]. This explains why black holes are common astrophysically, while wormholes have not been observed. However, the recently discovered acceleration of the universe implies that cosmic evolution is dominated by unknown dark energy, which violates at least the strong energy condition. Also, it is well known that negative energy densities are endemic in quantum field theory. Whatever the identity of the exotic matter, there is a unified framework for traversible wormholes, black holes and white holes [5]–[8]. Since the causal nature of the horizons depends on the sign of the energy density, which may change with time, black holes and wormholes should theoretically be interconvertible. A famous example is a Schwarzschild black hole evaporating by Hawking radiation, which semi-classically is a traversible wormhole by any reasonable definition [2, 5].

Returning to the Morris-Thorne criteria, the usability criteria (5)–(6) are already sufficiently general. However, any concrete study of wormhole dynamics requires (7) a specific exotic matter model, so that there are field equations to determine the evolution. Most early studies ignored the identity of the exotic matter [3, 4], but authors increasingly address this issue. Much work has concentrated on alternative gravitational theories such as scalar-tensor theories, where static wormhole solutions are found to be common. Other work draws the exotic matter from quantum field theory in a semi-classical approximation. The author has proposed using simple exotic matter models, in order to study issues of principle which were hardly understood. This has led to concrete examples which show conclusively that (8) wormhole stability depends on the exotic matter model. Although there have been various studies of linearization stability, dynamic stability has been investigated in only two cases: the HKL wormhole was found analytically to be dynamically stable [10], while the first Morris-Thorne wormhole was found numerically to be dynamically unstable [11]. Finally, concerning (9) how to assemble or construct a wormhole: irradiating a CGHS black hole with negative energy converts it to an HKL wormhole [10, 12], and similarly a wormhole discovered recently by the author [13] can be constructed from a Schwarzschild black hole [14]. The main results of these recent studies are summarized in the following.

3 Analytical results

The first concrete results were found using an exactly soluble model, CGHS two-dimensional dilaton gravity, generalized to include a massless ghost (negative-energy) Klein-Gordon field, which supports the existence of static wormhole solutions [10]. One can then set initial data corresponding to dynamical perturbations of a CGHS black hole or an HKL wormhole, then analytically find the evolved space-time. As described in last year's proceedings [2], solutions were found describing wormhole collapse to a black

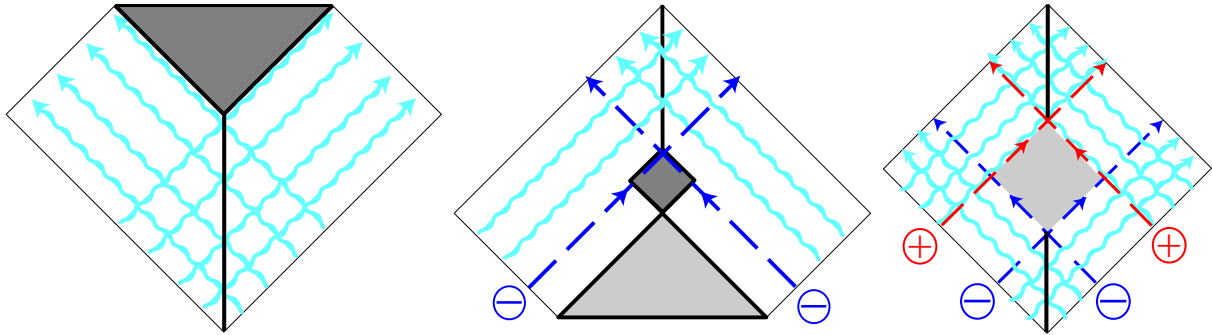


Figure 1: Penrose space-time diagrams. (i) A wormhole collapses to a black hole if the supporting ghost radiation, represented by wavy lines, is switched off. (ii) A wormhole is constructed from a black hole by irradiating with impulsive ghost radiation, represented by dashed lines, followed by constant non-impulsive radiation to support the resulting wormhole. (iii) A wormhole is enlarged by beaming in impulsive ghost radiation, switching off the non-impulsive radiation, then beaming in compensating impulsive normal radiation. The bold lines represent the trapping horizons, light shading indicates past trapped regions and darker shading indicates future trapped regions. The impulses discontinuously shift the trapping horizons constituting the black-hole horizons or the wormhole throat. In the dilaton gravity model, this refers to the CGHS black hole and the HKL wormhole, and in full Einstein gravity to the Schwarzschild black hole and a recently discovered wormhole.

hole, wormhole construction from a black hole, wormhole operation for transport, including the back-reaction of the transported matter on the wormhole, and wormhole maintenance, i.e. maintaining a static state under transportation [10].

In order to study similar dynamic processes in full Einstein gravity, the author proposed a simple exotic matter model, pure ghost radiation, i.e. pure radiation with negative energy density [2]. Wormhole solutions of the Morris-Thorne type were found [13], supported by equal left-moving and right-moving radiation. Since the radiation propagates without interaction or backscattering, it is easy to see what happens if it is suddenly switched off from both sides of the wormhole: the wormhole collapses to a black hole, as in Fig.1(i). The radiation escapes in Vaidya regions, leaving a vacuum Schwarzschild region, with continuity determining that the wormhole throat bifurcates to form the black-hole horizons.

For other dynamic processes, a useful idealization is impulsive radiation, where the radiation is concentrated so as to deliver finite energy and momentum in an instant. In the two-dimensional model, further examples were given of wormhole construction and operation, plus how to stably enlarge a wormhole [12]. Corresponding versions of some of these processes can be found analytically in full Einstein gravity by matching Schwarzschild, Vaidya and static-wormhole regions [14]. In particular, irradiating a Schwarzschild black hole with impulsive then constant ghost radiation can convert it to a wormhole of the above type, as in Fig.1(ii). Also, the wormhole can be enlarged by adding additional impulsive ghost radiation followed by balancing normal impulsive radiation, as in Fig.1(iii). The ghost radiation is switched off between the impulses, so that the middle region is found analytically as part of a Schwarzschild white hole, therefore expanding and increasing the wormhole size. The opposite ordering of the impulses would reduce the wormhole size. These will be the first analytic solutions describing wormhole construction or enlargement in full Einstein gravity.

4 Numerical results

The first numerical study of wormhole dynamics was also performed recently [11]. The starting point was a static wormhole which is best known as Morris & Thorne's opening example, though it is actually a solution for a massless ghost Klein-Gordon field, as was shown earlier by various authors. The idea was to study dynamical perturbations of this static wormhole, using the spherically symmetric Einstein system with the above exotic matter model, and also a normal massless Klein-Gordon field, to see the effect on the wormhole of normal matter, like an astronaut or spaceship traversing the wormhole.

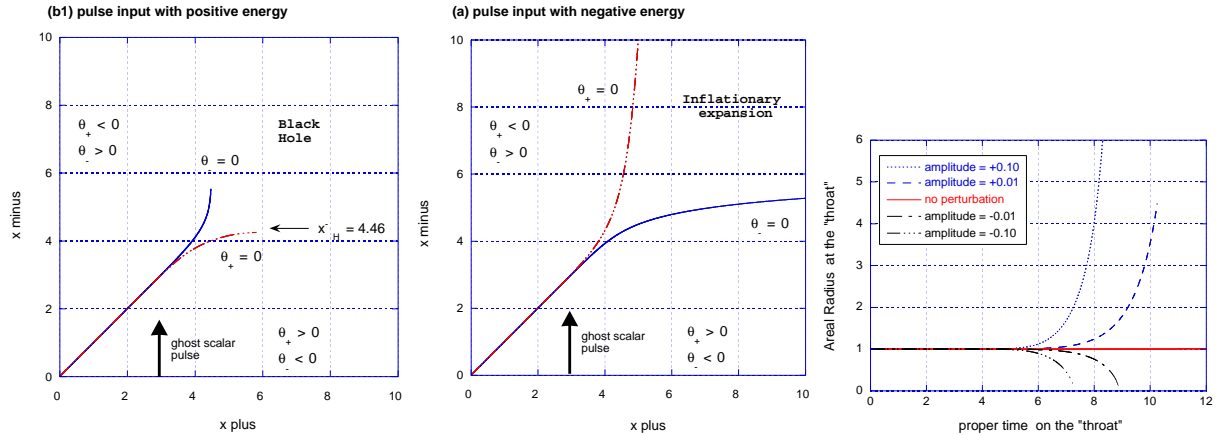


Figure 2: Location of the trapping horizons $\theta_{\pm} = 0$ when the static wormhole is perturbed by Gaussian pulses with (i) positive energy and (ii) negative energy. The axes are the dual-null coordinates x^{\pm} , so that a 45° counterclockwise rotation of the figure is a partial Penrose diagram. (iii) Areal radius of the throat $x^+ = x^-$, plotted as a function of proper time. Additional negative energy causes inflationary expansion, while reduced negative energy causes collapse to a black hole and central singularity. For smaller initial energy, the collapse or explosion occurs more slowly.

A numerical code was developed based on a dual-null coordinate system, in order to follow the horizon dynamics and radiation propagation accurately. After testing the code by evolving the static wormhole, the numerical experiments added or subtracted Gaussian pulses in the ghost field or the normal field, parametrized by amplitude, width and position. The space-time evolution was followed by tracking the trapping horizons and quantities such as area and energy. The local gravitational energy [7] is a key indicator; for the static wormhole it is positive, maximal at the throat and tends to zero at infinity, so that the total (Bondi or ADM) mass of the static wormhole vanishes. For dynamic perturbations, the sign and size of the initial Bondi energy E_0 was found to be the main factor affecting the outcome.

Fig.2(i) shows what happens for a positive-energy pulse, $E_0 > 0$: when the pulse hits the wormhole throat, the double trapping horizon bifurcates to form a tunnel of future trapped surfaces, with the two mouths accelerating away from each other and approaching the speed of light, forming the horizons of a black hole. The final mass M of the black hole can be easily extracted due to rapid convergence in the dual-null system. In summary, the wormhole has collapsed to a black hole.

Fig.2(ii) shows what happens for a negative-energy pulse, $E_0 < 0$: when the pulse hits the wormhole throat, the double trapping horizon bifurcates to form a tunnel of past trapped surfaces, with the two mouths accelerating away from each other and approaching the speed of light, forming the cosmological horizons of an inflationary universe. The Hubble constant H can be found by fitting the exponential curve of area against proper time, Fig.2(iii). In summary, the wormhole has exploded into an inflationary universe. This provides the first concrete mechanism for inflating wormholes from space-time foam to macroscopic size.

For Gaussian pulses in the normal field, the results are similar, except that the total energy E_0 is necessarily positive and so the wormhole always collapses to a black hole. Thus a traveller successfully crossing the wormhole would nevertheless look back to discover that the passage has caused the wormhole to collapse, thereby ironically sealing off the causal connection to the home universe.

By varying mainly the amplitude of the pulse, perturbations as small as $E_0 \approx \pm 10^{-4}a/2$ can be reliably evolved, where a is the initial throat radius. In all cases, positive or negative initial energy respectively causes collapse or explosion. Clearly the wormhole is dynamically unstable, though it had been previously found to be linearization stable, indicating a non-linear instability. For smaller perturbations, the collapse or explosion occurs more slowly and the final mass or Hubble constant is smaller. Unexpected critical behaviour was also discovered: the initial energy determines the collapse time $x - x_0 = -0.60 \ln E_0$ and there appears to be a minimal black-hole mass $M = 0.30a$ or Hubble constant $H = 1.1/a$ as perturbations tend to zero, for both exotic and normal field perturbations. All this is unexplained.

5 Conclusions

The largely new area of wormhole dynamics has been substantially developed recently, based on a local, dynamical theory of traversible wormholes [5], with mouths defined by temporal outer trapping horizons, unified with a local, dynamical theory of black holes [6]–[8]. Concrete examples [10]–[14] have supported the following conclusions.

- Traversable wormholes can be constructed from black holes by absorbing exotic matter.
- Traversable wormholes can collapse to black holes, by losing exotic matter or gaining normal matter, such as a traveller or spaceship.
- Traversable wormholes can explode to inflationary universes, by gaining exotic matter. This provides a mechanism for inflating wormholes from space-time foam to usable size.
- An exotic matter model must be specified for a given problem. Apart from semi-classical quantum field theory, dark energy models and alternative gravitational theories, simple models have theoretical merit in understanding basic principles.
- Traversable wormholes can be dynamically stable or unstable, depending on the exotic matter model. Linearization stability is not conclusive.
- Stable wormholes can be operated and maintained by balance of positive and negative energy, and enlarged or reduced by ordering of positive and negative energy.
- Numerically discovered critical behaviour suggests critical black-hole and inflationary-universe solutions.
- Wormhole dynamics with a specific exotic matter model need not be plagued by naked singularities or causal loops. Time-machine construction [3, 4], often described as ridiculously easy from (cut-and-paste) wormholes, is still an open question if given field equations are to be satisfied everywhere. Cosmic Censorship and Chronology Protection may yet survive physically reasonable exotic matter.

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